

Exercise 59

Table 4 shows the input, p , and output, q , for a linear function q . a. Fill in the missing values of the table. b. Write the linear function k .

p	0.5	0.8	12	b
q	400	700	a	1,000,000

Table 4

Solution

Begin by finding the equation of the line represented by this table. Use x for the input and y for the output so that the general equation of the line is

$$y = mx + b.$$

Two points on this line are $(0.5, 400)$ and $(0.8, 700)$. The first point says that when $x = 0.5$, $y = 400$.

$$400 = m(0.5) + b$$

The second point says that when $x = 0.8$, $y = 700$.

$$700 = m(0.8) + b$$

This is a system of two equations with two unknowns that can be solved.

$$\begin{cases} 0.5m + b = 400 \\ 0.8m + b = 700 \end{cases}$$

Subtract the respective sides of these equations to eliminate b .

$$0.5m - 0.8m = 400 - 700 \quad \rightarrow \quad -0.3m = -300 \quad \rightarrow \quad m = 1000$$

Multiply both sides of the first equation by 8, multiply both sides of the second equation by -5 ,

$$\begin{cases} 4m + 8b = 3200 \\ -4m - 5b = -3500 \end{cases}$$

and add the respective sides to eliminate m .

$$8b + (-5b) = 3200 + (-3500) \quad \rightarrow \quad 3b = -300 \quad \rightarrow \quad b = -100$$

Now that m and b have been solved for, the line is known.

$$y = 1000x - 100$$

When the input is 12, the output is

$$a = 1000(12) - 100 = 11\,900.$$

When the output is 1,000,000, the input is

$$1\,000\,000 = 1000b - 100$$

$$1\,000\,100 = 1000b$$

$$b = \frac{1\,000\,100}{1000} = 1000.1.$$